# Mechanical Properties of Solids and Fluids

## Fill in the Blanks

**Q.1. A wire of length L and cross sectional area A is made of a material of Young's modulus Y. If the wire is stretched by an amount x, the work done is ................. (1987 - 2 Marks)**

**Ans.** $\frac{YAx^2}{2L}$

**Solution.**

$$W = \frac{1}{2} \times Y \times (\text{strain})^2 \times Yd = \frac{1}{2} \times Y \times \frac{x^2}{L^2} \times AL = \frac{YAx^2}{2L}$$

**Q.2. A solid sphere of radius R made of a material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless pistion of area A floats on the surface of the liquid.**

**When a mass M is placed on the piston to compress the liquid the fractional change in the radius of the sphere, δ R/R, is ........... (1988 - 2 Mark)**

**Ans.** Mg/3Ak

**Solution.**

$$K = \frac{-\Delta P}{\Delta V / V}$$

where $\Delta P = \frac{Mg}{A}$ $\quad \therefore \quad -\frac{\Delta V}{V} = \frac{Mg}{AK}$

$$\Rightarrow -\frac{(V_f - V_i)}{V_i} = \frac{Mg}{AK} \quad \Rightarrow \frac{V_i - V_f}{V_i} = \frac{Mg}{AK}$$

$$\Rightarrow \frac{\frac{4}{3}\pi R^3 - \frac{4}{3}\pi(R - \delta R)^3}{\frac{4}{3}\pi R^3} = \frac{Mg}{AK}$$

$$\Rightarrow \frac{R^3 - [R^3 - 3R^2\delta R]}{R^3} = \frac{Mg}{AK} \quad \Rightarrow \frac{\delta R}{R} = \frac{Mg}{3AK}$$

**Q.3. A piece of metal floats on mercury. The coefficients of volume expansion of the metal and mercury are $\lambda_1$ and $\lambda_2$ respectively. If the temperatures of both mercury and the metal are increased by an amount $\Delta T$ , the fraction of the volume of the metal submerged in mercury changes by the factor .................. (1991 - 2 Mark)**

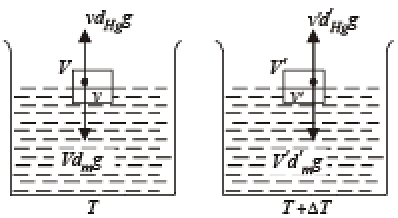
**Ans.** $(\lambda_2 - \lambda_1)\Delta T$

**Solution.** KEY CONCEPT

Using the relation for floatation, $vd_{Hgg} = Vd_m g$

Fraction of volume of metal submerged in mercury

$$= \frac{v}{V} = \frac{d_m}{d_{Hg}} = K_1 \text{ (say)}$$




In second case, when temperature is increased by $\Delta T$.

$$v'd'_{Hg}g = V'd'_m g$$

$$\Rightarrow \frac{v'}{V'} = \frac{d'_m}{d'_{Hg}}$$

= Fraction of volume of metal submerged in mercury = $K_2$ (say)

$$\therefore \frac{K_2}{K_1} = \frac{d'_m \times d_{Hg}}{d'_{Hg} \times d_m} = \frac{d'_m \times d'_{Hg}(1+\gamma_2\Delta T)}{d'_{Hg} \times d'_m(1+\gamma_1\Delta T)} = \frac{(1+\gamma_2\Delta T)}{(1+\gamma_1\Delta T)}$$

$$= (1+\gamma_2\,\Delta T)\,(1+\gamma_1\,\Delta T)^{-1}$$

$$= (1+\gamma_2\,\Delta T)\,(1-\gamma_1\,\Delta T) = 1+(\gamma_2-\gamma_1)\,\Delta T$$

**Note :** If $\gamma_2 - \gamma_1$ then $k_2 > k_1$

i.e., metal block will get immersed deeper

If $\lambda_2 < \lambda_1$ then $k_2 < k_1$

i.e. metal block will rise a bit as compared to its previous position.

$$\frac{K_2}{K_1}-1=(\gamma_2-\gamma_1)\Delta T \Rightarrow \frac{K_2-K_1}{K_1}=(\gamma_2-\gamma_1)\Delta T$$

**Q.4. A horizontal pipeline carries water in a streamline flow.**

**At a point along the pipe, where the cross- sectional area is 10 cm$^2$, the water velocity is 1 ms$^{-1}$ and the pressure is 2000 Pa. The pressure of water at another point where the cross-sectional area is 5 cm$^2$, is... Pa.**

**(Density of water = 10$^3$ kg.m$^{-3}$) (1994 - 2 Marks)**

**Ans.** 500 Pa.

**Solution. KEY CONCEPT**

Applying equation of continuity at cross section 1 and 2

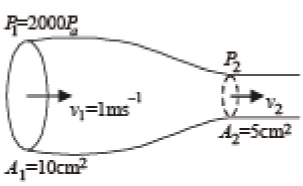
$A_1v_1 = A_2v_2 \Rightarrow 10 \times 1 = 5 \times v_2 \Rightarrow v_2 = 2m/s$

Applying Bernoulli's theorem

$$P_1+\frac{1}{2}\rho v_1^2 = P_2+\frac{1}{2}\rho v_2^2$$

$$\Rightarrow 2000+\frac{1}{2}\times 1000\times 1^2 = P_2+\frac{1}{2}\times 1000\times 2^2$$

$$\Rightarrow P_2 = 500\ Pa$$



## True/False

**Q.1. A man is sitting in a boat which is floating in a pond. If the man drinks some water from the pond, the level of the water in the pond decreases.**

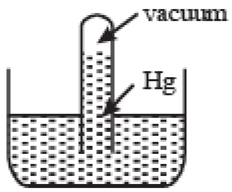
**Ans.** F

**Solution.** When the man drinks some water from the pond, his weight increases and therefore the boat will sink further. The further sinking of the boat will displace the same volume of water in pond as drunk by man. Therefore, there will no change in the level of water in the pond.

**Q.2. A barometer made of a very narrow tube (see Fig) is placed at normal temperature and pressure. The coefficient of volume expansion of mercury is**

**0.00018 per C° and that of the tube is negligible. The temperature of mercury in the barometer is now raised by 1°C, but the temperature of the atmosphere does not change.**

**Then the mercury height in the tube remains unchanged. (1983 - 2 Marks)**



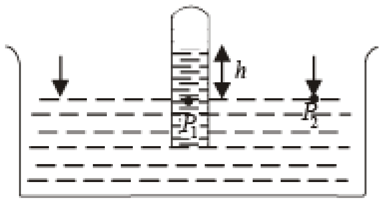

**Ans.** F

**Solution.** Pressure $P_1 = P_2 = 1$ atm $= h\rho g$

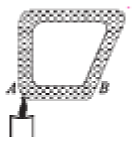
**Note :** On changing the temperature, g will not change and atmospheric pressure will not change.

$\therefore h \times r =$ constant.

When temperature is increased, the density of Hg decreases and hence, h increases.




**Q.3. Water in a closed tube (see Fig) is heated with one arm vertically placed above a lamp. Water will begin to circulate along the tube in counter-clockwise direction. (1983 - 2 Marks)**




**Ans. F**

**Solution.** When water is heated at end A, the density decreases and the water moves up. This is compensated by the movement of water from B to A i.e., in clockwise direction.

**Q.4. A block of ice with a lead shot embedded in it is floating on water contained in a vessel. The temperature of the system is maintained at 0°C as the ice melts. When the ice melts completely the level of water in the vessel rises. (1986 - 3 Marks)**

**Ans. F**

**Solution.** When the block of ice melts, the lead shot will ultimately sink in the water. When lead shot sinks, it will displace water equal to its own volume. But when lead shot was embedded in ice, it displaced more volume of water than its own volume because $d_{lead} > d_{water}$. Therefore, level of water will fall.

**Q.1. A column of mercury of 10 cm length is contained in the middle of a narrow horizontal 1 m long tube which is closed at both the ends. Both the halves of the tube contain air at a pressure of 76 cm of mercury. By what distance will the column of mercury be displaced if the tube is held vertically? (1978)**

**Ans.** 2.95 cm

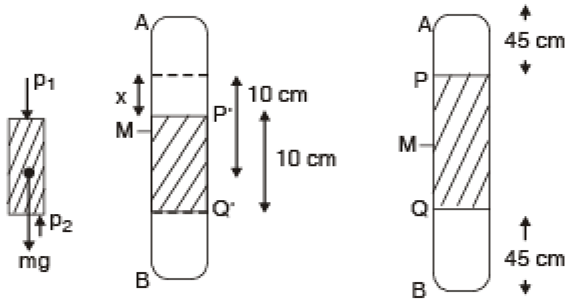
**Solution.** M is the mid-point of tube AB.

At equilibrium

$p_1 \times A + mg = p_2 \times A$

$p_1 \times A + 10 \times A \times d_{Hg}g = p_2 \times A$

$\Rightarrow p_1 + 10d_{Hg} \times g = p_2$ ... (i)




For air present in column AP

$p \times 45 \times A = p_1 \times (45 + x) \times A$

$$\Rightarrow p_1 = \frac{45}{45+x} \times 76d_{Hg} \times g \quad \text{...(ii)}$$

For air present in column QB

$p \times 45 \times A = p_2 \times (45 - x) \times A$

$$\Rightarrow p_2 = \frac{45}{45-x} \times 76d_{Hg} \times g \quad \text{...(iii)}$$

From (i), (ii) and (iii)

$$\frac{45\times 76\times d_{Hg}g}{45+x}+10d_{Hg}\times g=\frac{45}{45-x}\times 76\times d_{Hg}\times g$$

$$\Rightarrow \frac{45\times 76}{45+x}+10=\frac{45\times 76}{45-x}$$

$x = 2.95$ cm

**Q.2. A point mass m is suspended at the end of a massless wire of length l and cross section A. If Y is the Young's modulus for the wire, obtain the frequency of oscillation for the simple harmonic motion along the vertical line. (1978)**

**Ans.** $\frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$

**Solution.** From fig. (b), due to equilibrium

T = mg ... (i)

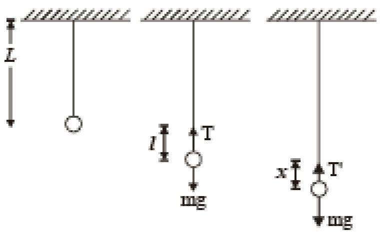



Fig. (a) Fig. (b) Fig. (c)

But $Y=\frac{T/A}{\ell/L}$

$\Rightarrow T=\frac{YA\ell}{L}$ ... (ii)

From (i) and (ii)

$mg=\frac{YA\ell}{L}$ ... (iii)

From fig. (c) Restoring force

$$=-[T'-mg]=-\left[\frac{YA(\ell+x)}{L}-\frac{YA\ell}{L}\right] \quad \text{[from (iii)]}$$

$$=\frac{-YAx}{L}$$

On comparing this equation with F = – m$\omega^2$x, we get

$$m\omega^2 = \frac{YA}{L} \Rightarrow \omega = \sqrt{\frac{YA}{mL}} \Rightarrow \frac{2\pi}{T} = \sqrt{\frac{YA}{mL}}$$

$$\text{Frequency } f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$$

**Q.3. A cube of wood supporting 200 gm mass just floats in water. When the mass is removed, the cube ruses by 2cm. What is the size of the cube? (1978)**

**Ans.** l = 10 cm

**Solution. Let the edge of cube be ℓ. When mass is on the cube of wood**

$$200g + \ell^3 d_{wood} g = \ell^3 d_{water} g$$

$$\Rightarrow \ell^3 d_{wood} = \ell^3 d_{water} - 200 \quad \text{...(i)}$$

When the mass is removed

$$\ell^3 d_{wood} = (\ell - 2)\ell^2 d_{water} \quad \text{...(ii)}$$

From (i) and (ii)

$$\ell^3 d_{water} - 200 = (\ell - 2)\ell^2 d_{water}$$

$$\text{But } d_{water} = 1$$

$$\therefore \ell^3 - 200 = \ell^2(\ell - 2) \Rightarrow \ell = 10\text{cm}$$

**Q.4. A boat floating in a water tank is carrying a number of large stones. If the stones are unloaded into water, what will happen to the water level? (1979)**

**Ans.** Fall

**Solution.** KEY CONCEPT :

When the stones were in the boat, the weight of stones were balanced by the buoyant force.

$$V_s d_s = V_\ell d_\ell$$

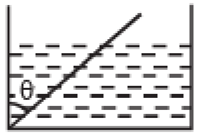
$V_\ell$, $V_s$ = volume of liquid and stone respectively

$d_\ell$, $d_s$ = density of liquid and stone respectively

Since, $d_s > d_\ell$ $\therefore V_s < V_\ell$

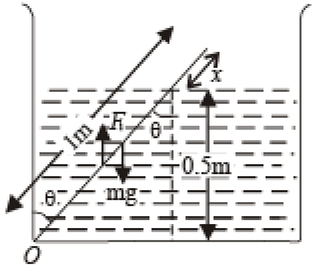
Therefore when stones are put in water, the level of water falls.

**Q.5. A wooden plank of length 1 m and uniform cross-section is hinged at one end to the bottom of a tank as shown in fig The tank is filled with water upto a height 0.5 m. The specific gravity of the plank is 0.5. Find the angle θ that the plank makes with the vertical in the equilibrium position. (Exclude the case θ = 0°) (1984- 8 Marks)**




**Ans.** 45°

**Solution.** For equilibrium $F_{net} = 0$ and $\tau_{net} = 0$




Taking moment about O

$$mg \times \frac{\ell}{2} \sin\theta = F_T\left(\frac{\ell - x}{2}\right)\sin\theta \qquad \text{...(i)}$$

Also $F_T$ = wt. of fluid displaced = $[(\ell - x) A] \times \rho_w g$... (ii)

And $m = (\ell A)\, 0.5\rho_w$ ... (iii)

Where A is the area of cross section of the rod.

From (i), (ii) and (iii)

$$(\ell A)0.5\rho_w g\times\frac{\ell}{2}\sin\theta=[(\ell-x)A]\rho_w g\times\left(\frac{\ell-x}{2}\right)\sin\theta$$

Here, $\ell = 1$ m

$\therefore (1 - x)^2 = 0.5 \quad \Rightarrow x = 0.293$ m

From the diagram

$$\cos\theta=\frac{0.5}{1-x}=\frac{0.5}{0.707}\quad\Rightarrow\quad\theta=45°$$

**Q.6. A ball of density d is dropped on to a horizontal solid surface. It bounces elastically from the surface and returns to its original position in a time $t_1$. Next, the ball is released and it falls through the same height before striking the surface of a liquid of density of $d_L$ (1992 - 8 Marks)**

**(a) If $d < d_L$, obtain an expression (in terms of d, $t_1$ and $d_L$) for the time $t_2$ the ball takes to come back to the position from which it was released. (b) Is the motion of the ball simple harmonic?**

**(c) If $d = d_L$, how does the speed of the ball depend on its depth inside the liquid? Neglect all frictional and other dissipative forces. Assume the depth of the liquid to be large.**

**Ans.** (a) $\frac{d_L t_1}{d_L - d}$

**(b)** no

**(c)** remains same

**Solution.** (a) Let the ball be dropped from a height h. During fall

$$V=ut+at=0+g\frac{t_1}{2}\ \Rightarrow\ t_1=\frac{2v}{g}$$

In the second case the ball is made to fall through the same height and then the ball strikes the surface of liquid of density $d_L$. When the ball reaches inside the liquid, it is under the influence of two force (i) Vdg, the weight of ball in downward direction

(ii) $Vd_Lg$, the upthrust in upward direction.

**Note :** The viscous forces are absent. (given)

Since, $d_L > d$

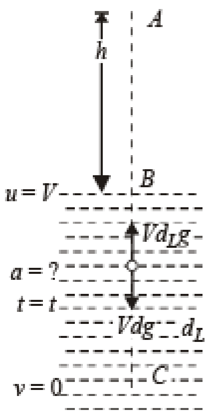
the upward force is greater and the ball starts retarding.

**For motion B to C**

u = V , v = 0, t = t, a = – a

v = u + at ⇒ 0 = v + (-a)t

⇒ t = v/a




$$\text{Now, } a = \frac{F_{net}}{m}$$

$$= \frac{Vd_Lg - Vdg}{Vd} = \frac{(d_L - d)g}{d}$$

$$\Rightarrow \quad t = \frac{vd}{(d_L - d)g} \qquad \text{... (iii)}$$

Therefore,

$$t_2 = t_1 + 2t = t_1 + \frac{2dv}{(d_L - d)g}$$

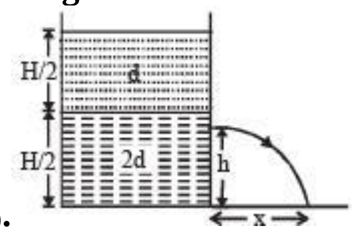
$$= t_1 + \frac{2d}{(d_L - d)g}\,\frac{t_1 g}{2} = t_1\left[1 + \frac{d}{d_L - d}\right]$$

$$\Rightarrow \quad t_2 = \frac{d_L t_1}{d_L - d}$$

(b) Since the retardation is not proportional to displacement, the motion of the ball is not simple harmonic.

(c) If $d = d_L$ then the retardation $a = 0$. Since the ball strikes the water surface with some velocity, it will continue with the same velocity in downward direction (until it is interrupted by some other force).

**Q.7. A container of large uniform cross-sectional area A resting on a horizontal surface, holds two immiscible, non-viscous and incompressible liquids of densities d and 2d, each of height H/2 as shown in the figure. The lower density liquid is open to the atmosphere having pressure $P_0$.**




**(a) A homogeneous solid cylinder of length L(L < H/2), cross-sectional area A/5 is immersed such that it floats with its axis vertical at the liquid-liquid interface with length L/4 in the denser liquid. Determine:**

**(i) the density D of the solid and**

**(ii) the total pressure at the bottom of the container.**

**(b) The cylinder is removed and the original arrangement is restored. A tiny hole of area s(s << A) is punched on the vertical side of the container at a height h(h < H/2).**

**Determine :**

**(i) the initial speed of efflux of the liquid at the hole,**

**(ii) the horizontal distance x travelled by the liquid initially, and**

**(iii) the height $h_m$ at which the hole should be punched so that the liquid travels the maximum distance $x_m$ initially. Also calculate $x_m$.**

**(Neglect the air resistance in these calculations.)**

**Ans.**

(a) (i) $\frac{5d}{4}$ (ii) $P_0 + \left(\frac{3H}{2} + \frac{L}{4}\right) dg$

(b) (i) $\frac{\sqrt{3H-4h}}{2} g$ (ii) $\sqrt{(3H-4h)h}$ (iii) $\frac{3H}{8}, \frac{3H}{4}$

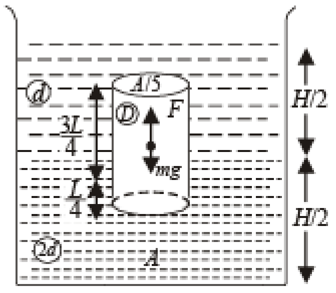
**Solution. (a) (i) KEY CONCEPT :**

Since the cylinder is in equilibrium in the liquid therefore Weight of cylinder = upthrust

$mg = F_{T_1} + F_{T_2}$ where

$F_{T_1}$ and $F_{T_2}$ :

= upth rust due to lower a n d upper liquid respectively




$$\frac{A}{5} \times L \times D \times g = \frac{A}{5} \times \frac{L}{4} \times 2d \times g + \frac{A}{5} \times \frac{3L}{4} \times d \times g$$

$$\Rightarrow \quad D = \frac{2d}{4} + \frac{3d}{4} = \frac{5d}{4}$$

(ii) Total pressure at the bottom of the cylinder = Atmospheric pressure + Pressure due to liquid of density d + Pressure due to liquid of density 2d + Pressure due to cylinder [Weight/Area]

$$P = P_0 + \frac{H}{2}dg + \frac{H}{2} \times 2d \times g + \frac{\frac{A}{5} \times L \times D \times g}{A}$$

$$\Rightarrow \quad P = P_0 + \left(\frac{3H}{2} + \frac{L}{4}\right)dg \quad \left[\because D = \frac{5d}{4}\right]$$

**(b) KEY CONCEPT :**

Applying Bernoulli's theorem

$$P_0 + \left[\frac{H}{2} \times d \times g + \left(\frac{H}{2} - h\right) 2d \times g\right]$$

$$= P_0 + \frac{1}{2}(2d)v^2 \Rightarrow v = \sqrt{\frac{(3H - 4h)}{2} g}$$

Horizontal Distance x

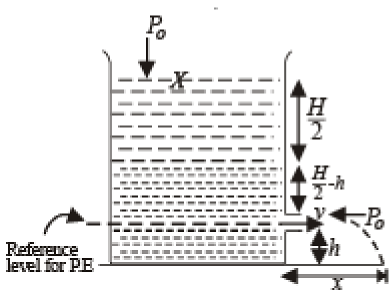
$u_x = v$; $t = t$; $x = vt$ .... (i)

For vertical motion of liquid falling from hole

$u_y = 0$, $S_y = h$, $a_y = g$, $t_y = t$

$$S = ut + \frac{1}{2}at^2$$

$$\Rightarrow \quad h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}} \qquad \text{...(ii)}$$




From (i) and (ii)

$$x = v_y \times \sqrt{\frac{2h}{g}} = \sqrt{(3H - 4h)\frac{g}{2}} \times \sqrt{\frac{2h}{g}}$$

$$= \sqrt{(3H - 4h)h} \qquad \text{... (iii)}$$

For finding the value of h for which x is maximum, we differentiate equation (iii) w.r.t. t

$$\frac{dx}{dt} = \frac{1}{2}[3H - 4h)h]^{-1/2}\{3H - 8h\}$$

Putting dx/dt = 0 for finding values h for maxima/minima

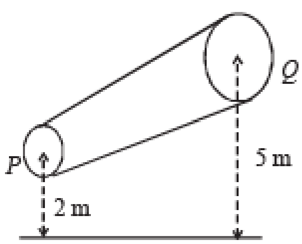
$$\frac{1}{2}[(3H - 4h)]^{-1/2}[3H - 8h] = 0 \Rightarrow h = \frac{3H}{8}$$

$$\therefore \quad x_m = \sqrt{\left[3H - 4\left(\frac{3H}{8}\right)\right]\frac{3H}{8}} \qquad \text{[From (iii)]}$$

$$= \sqrt{\frac{12H}{8} \times \frac{3H}{8}} = \frac{6H}{8} = \frac{3H}{4}$$

**Q.8. A non-viscous liquid of constant density 1000 kg/m$^3$ flows in a streamline motion along a tube of variable cross section.**

**The tube is kept inclined in the vertical plane as shown in Figure. The area of cross section of the tube two points P and Q at heights of 2 metres and 5 metres are respectively 4×10$^{-3}$m$^2$ and 8×10$^{-3}$ m$^2$. The velocity of the liquid at point P is 1 m/s. Find the work done per unit volume by the pressure and the gravity forces as the fluid flows from point P to Q. (1997 - 5 Marks)**




**Ans.** 29.025 × 10$^3$ J/m$^3$ ; 29.4 × 10$^3$ J/m.

**Solution.** Given that

$\rho$ = 1000 kg/m$^3$, $h_1$ = 2m, $h_2$ = 5 m

$A_1$ = 4 × 10$^{-3}$m$^2$, $A_2$ = 8 × 10$^{-3}$ m$^2$, $v_1$ = 1 m/s

Equation of continuity

$$A_1v_1 = A_2v_2 \quad \therefore v_2 = \frac{A_1v_1}{A_2} = 0.5\,\text{m/s}$$

According to Bernoulli's theorem,

$$(p_1 - p_2) = \rho g (h_2 - h_1) - \frac{1}{2}\rho(v_2^2 - v_1^2)$$

Where $(p_1 - p_2)$ = work done/vol. [by the pressure]

$\rho g (h_2 - h_1)$ = work done/vol. [by gravity forces]
Now, work done/vol. by gravity forces

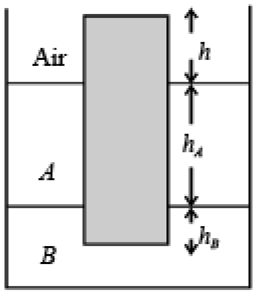
$= \rho g (h_2 - h_1) = 10^3 \times 9.8 \times 3 = 29.4 \times 10^3$ J/m$^3$.

$$\text{And } \frac{1}{2}\rho(v_2^2 - v_1^2) = \frac{1}{2} \times 10^3 \left[\frac{1}{4} - 1\right] = -\frac{3}{8} \times 10^3 \text{ J/m}^3$$
$$= -0.375 \times 10^3 \text{ J/m}^3$$

∴ Work done / vol. by pressure

$= 29.4 \times 10^3 - 0.375 \times 10^3$ J/m$^3$ = 29.025 × 103 J/m$^3$.

**Q.9. A unifor m solid cylinder of density 0.8 g/cm$^3$ floats in equilibrium in a combination of two non-mixing liquids A and B with its axis vertical.**




**The densities of the liquids A and B are 0.7 g/cm$^3$ and 1.2 g/ cm$^3$, respectively. The height of liquid A is $h_A$ = 1.2 cm. The length of the part of the cylinder immersed in liquid B is $h_B$= 0.8 cm.**

**(a) Find the total force exerted by liquid A on the cylinder.**

**(b) Find h, the length of the part of the cylinder in air.**

**(c) The cylinder is depressed in such a way that its top surface is just below the upper surface of liquid A and is then released. Find the acceleration of the cylinder immediately after it is released.**

**Ans.** (a) zero (b) 0.25 cm (c) g/6,↑

**Solution.** (a) As the pressure exerted by liquid A on the cylinder is radial and symmetric, the force due to this pressure cancels out and the net value is zero.

(b) For equilibrium, Buoyant force = weight of the body

$$\Rightarrow h_A \rho_A A g + h_B \rho_B A g = (h_A + h + h_B) A \rho_C g$$

(where $\rho_C$ = density of cylinder)

$$h = \left(\frac{h_A \rho_A + h_B \rho_B}{\rho_C}\right) - (h_A + h_B) = 0.25 \text{ cm}$$

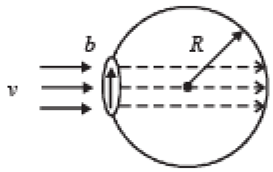
(c) $$a = \frac{F_{\text{Buoyant}} - Mg}{M}$$

$$= \left[\frac{h_A \rho_A + \rho_B (h + h_B) - (h + h_A + h_B)\rho_C}{\rho_C (h + h_A + h_C)}\right] g$$

$$= \frac{g}{6} \text{ upwards}$$

**Q.10. A bubble having surface tension T and radius R is formed on a ring of radius b (b << R). Air is blown inside the tube with velocity v as shown. The air molecule collides perpendicularly with the wall of the bubble and stops.**

**Calculate the radius at which the bubble separates from the ring. (2003 - 4 Marks)**



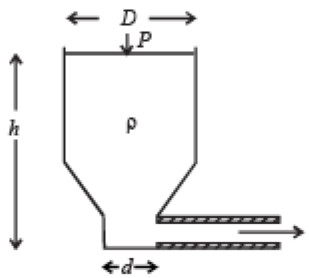

**Ans.** $\frac{4T}{\rho v^2}$

**Solution. KEY CONCEPT :** When the force due to excess pressure in the bubble equals the force of air striking at the bubble, the bubble will detach from the ring.

$$\therefore \rho A v^2 = \frac{4T}{R} \times A \Rightarrow R = \frac{4T}{\rho v^2}$$

**Q.11. Shown in the figure is a container whose top and bottom diameters are D and d respectively. At the bottom of the container, there is a capillary tube of outer radius b and inner radius a.**

**The volume flow rate in the capillary is Q. If the capillary is removed the liquid comes out with a velocity of $v_0$. The density of the liquid is given as ρ. Calculate the coefficient of viscosity η. (2003 - 4 Marks)**




**Ans.** $\frac{\pi}{8Q\ell} \times \frac{1}{2}\rho v_0^2 \left[1 - \frac{d^4}{D^4}\right] \times a^4$

**Solution. KEY CONCEPT :** When the tube is not ther e, using Bernoulli's theorem

$$P + P_0 + \frac{1}{2}\rho v_1^2 + \rho g H = \frac{1}{2}\rho v_0^2 + P_0$$

$$\Rightarrow \; P + \rho g H = \frac{1}{2}\rho(v_0^2 - v_1^2)$$

But according to equation of continuity

$$v_1 = \frac{A_2 v_0}{A_1}$$

$$\therefore \; P + \rho g H = \frac{1}{2}\rho\left[v_0^2 - \left(\frac{A_2}{A_1}v_0\right)^2\right]$$

$$P + \rho g H = \frac{1}{2}\rho v_0^2\left[1 - \left(\frac{A_2}{A_1}\right)^2\right]$$

Here, P + ρgH = ΔP According to Poisseuille's equation

$$Q = \frac{\pi(\Delta P)a^4}{8\eta l} \Rightarrow \eta = \frac{\pi(\Delta P)a^4}{8Q\ell}$$

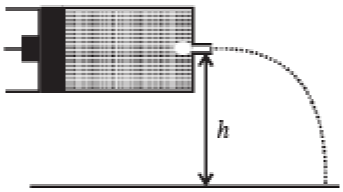
$$\therefore \quad \eta = \frac{\pi(P+\rho gH)a^4}{8Q\ell} = \frac{\pi}{8Q\ell} \times \frac{1}{2}\rho v_0^2\left[1-\left(\frac{A_2}{A_1}\right)^2\right] \times a^4$$

Where $\frac{A_2}{A_1} = \frac{d^2}{D^2}$

Where $\frac{A_2}{A_1} = \frac{d^2}{D^2}$

$$\eta = \frac{\pi}{8Ql} \times \frac{1}{2}\rho v_0^2\left[1-\frac{d^4}{D^4}\right] \times a^4$$

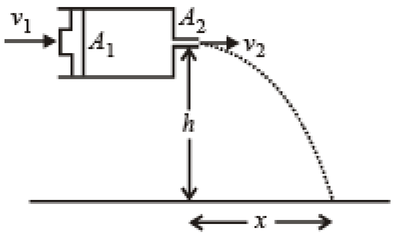
**Q.12. A tube has two area of cross-section s as shown in figure. The diameters of the tube are 8 mm and 2 mm. Find range of water falling on horizontal surface, if piston is moving with a constant velocity of 0.25 m/s, h = 1.25 m (g = 10 m/s²) (2004 - 2 Marks)**




**Ans.** 2m

**Solution.** From law of continuity $A_1v_1 = A_2v_2$

Given $A_1 = \pi \times (4 \times 10^{-3}\ m)^2$, $A_2 = \pi \times (1 \times 10^{-3}\ m)^2$




$$v_1 = 0.25\ m/s$$

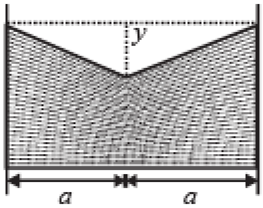
$$\therefore \quad v_2 = \frac{\pi \times (4 \times 10^{-3})^2 \times 0.25}{\pi \times (1 \times 10^{-3})^2} = 4\ m/s$$

Also, $h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$

Horizontal range $x = v_2 \times t = v_2\sqrt{\frac{2h}{g}} = 4 \times \sqrt{\frac{2 \times 1.25}{10}} = 2\text{m}$

**Q.13. A uniform wire having mass per unit length λ is placed over a liquid surface. The wire causes the liquid to depress by y(y << a) as shown in figure. Find surface tension of liquid.**

**Neglect end effect. (2004 - 2 Marks)**



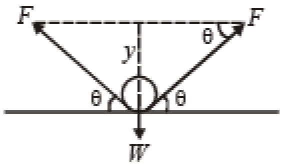

**Ans.** $\frac{\lambda ag}{2y}$

**Solution.** The free body diagram of wire is given below. If ℓ is the length of wire, then for equilibrium 2F sin θ = W.

**F = S × ℓ**

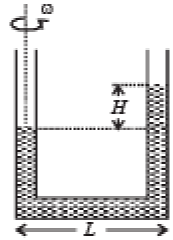
or, $2S \times \ell \times \sin\theta = \lambda \times \ell \times g$

or, $S = \frac{\lambda g}{2\sin\theta}$




$\therefore \quad S = \frac{\lambda g}{2y/a} = \frac{a\lambda g}{2y} \qquad \left[\because \sin\theta = \frac{y}{a}\right]$

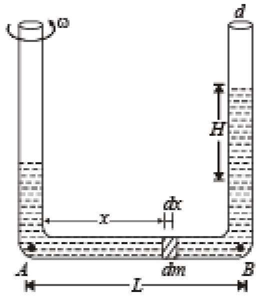
**Q.14. A U tube is rotated about one of it's limbs with an angular velocity ω. Find the difference in height H of the liquid (density ρ) level, where diameter of the tube d << L. (2005 - 2 Marks)**




**Ans.** $H = \frac{\omega^2 L^2}{2g}$

**Solution. Weight of liquid of height H**

$$= \frac{\pi d^2}{4} \times H \times \rho \times g \quad ...(i)$$




**Let us consider a mass dm situated at a distance x from A as shown in the figure. The centripetal force required for the mass to rotate = (dm) xw²**

∴ The total centripetal force required for the mass of length L to rotate

$$= \int_0^L (dm)\, x\omega^2 \quad \text{where } dm = \rho \times \frac{\pi d^2}{4} \times dx$$

$$\therefore \quad \text{Total centripetal force} = \int_0^L \left(\rho \times \frac{\pi d^2}{4} \times dx\right) \times \left(x\omega^2\right)$$

$$= \rho \times \frac{\pi d^2}{4} \times \omega^2 \int_0^L x\,dx$$

$$= \rho \times \frac{\pi d^2}{4} \times \omega^2 \times \frac{L^2}{2} \qquad \text{...(ii)}$$

This centripetal force is provided by the weight of liquid of height H.

From (i) and (ii)

$$\frac{\pi d^2}{4} \times H \times \rho \times g = \rho \times \frac{\pi d^2}{4} \times \frac{\omega^2 \times L^2}{2} \Rightarrow H = \frac{\omega^2 L^2}{2g}$$

# Match the Following

**DIRECTIONS (Q. No. 1) : Following question has matching lists. The codes for the lists have choices (a), (b), (c) and (d) out of which ONLY ONE is correct.**

**Q.1. A person in lift is holding a water jar, which has a small hole at the lower end of its side. When the lift is at rest, the water jet coming out of the hole hits the floor of the lift at a distance d of 1.2 m from the person. In the following, state of the lift's motion is given in List-I and the distance where the water jet hits the floor of the lift is given in List-II. Match the statements from List-I with those in List-II and select the correct answer using the code given below the lists. (JEE Adv. 2014)**

| **List - I** | **List - II** |
|---|---|
| **P.** Lift is accelerating vertically up | **1.** $d = 1.2$ m |
| **Q.** Lift is accelerating vertically down with an acceleration less than the gravitational acceleration | **2.** $d > 1.2$ m |
| **R.** Lift is moving vertically up with constant speed | **3.** $d < 1.2$ m |
| **S.** Lift is falling freely | **4.** No water leaks out of the jar |

**Code:**

**(a) P-2, Q-3, R-2, S-4**

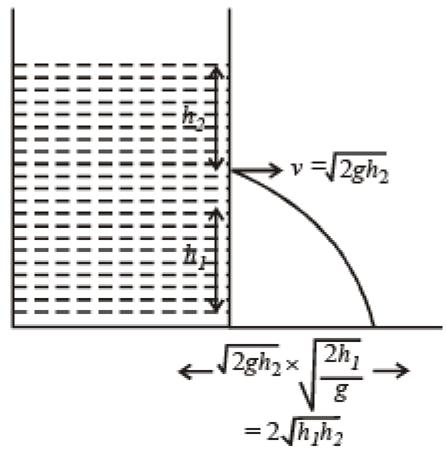
**(b) P-2, Q-3, R-1, S-4**

**(c) P-1, Q-1, R-1, S-4**

**(d) P-2, Q-3, R-1, S-1**

**Ans.** (c)

**Solution.**




If $g_{eff} > g$

$g_{eff} = g$

$g_{eff} < g$

In all the three cases

$d = 2\sqrt{h_1h_2} = 1.2\text{ m}$

If geff = 0, then no water leaks out

**Integer Value Correct Type**

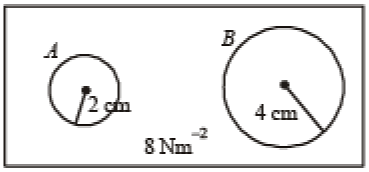
**Q.1. Two soap bubbles A and B are kept in a closed chamber where the air is maintained at pressure 8 N/m$^2$. The radii of bubbles A and B are 2 cm and 4 cm, respectively. Surface tension of the soap-water used to make bubbles is 0.04 N/m.**

**Find the ratio $n_B/n_A$, where $n_A$ and $n_B$ are the number of moles of air in bubbles A and B, respectively. [Neglect the effect of gravity.] (2009)**

**Ans.** 6

**Solution. For bubble A :**

If PA is the pressure inside the bubble then




$$P_A - 8 = \frac{4T}{R_A} = \frac{4 \times 0.04}{0.02} = 8 \Rightarrow P_A = 16 N/m^2$$

According to ideal gas equation,

$$P_A V_A = n_A R T_A \Rightarrow 16 \times \frac{4}{3}\pi(0.02)^3 = n_A R T_A \;\dots \text{(i)}$$

**For bubble B :**

If PB is the pressure inside the bubble then

$$P_B - 8 = \frac{4T}{R_B} = \frac{4 \times 0.04}{0.04} = 4 \Rightarrow P_B = 12 N/m^2$$

According to ideal gas equation

$$P_B V_B = n_B R T_B \Rightarrow 12 \times \frac{4}{3}\pi(0.04)^3 = n_B R T_B \dots \text{(ii)}$$

Dividing (ii) by (i) we get

$$\frac{12 \times \frac{4}{3}\pi(0.04)^3}{16 \times \frac{4}{3}\pi(0.02)^3} = \frac{n_B}{n_A} \;[\because T_A = T_B]$$

$$\therefore \quad \frac{n_B}{n_A} = 6$$

**Q.2. A cylindrical vessel of height 500 mm has an orifice (small hole) at its bottom. The orifice is initially closed and water is filled in it up to height H. Now the top is completely sealed with a cap and the orifice at the bottom is opened. Some water comes out from the orifice and the water level in the vessel becomes steady with height of water column being 200 mm. Find the fall in height (in mm) of water level due to opening of the orifice.**

**[Take atmospheric pressure = 1.0 × $10^5$ N/$m^2$, density of water = 1000 kg/$m^3$ and g = 10 m/$s^2$. Neglect any effect of surface tension.]**

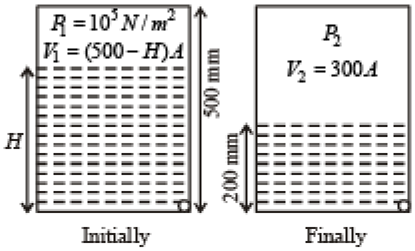
**Ans.** 6

**Solution.** Initially, the pressure of air column above water is $P_1 = 10^5$ $Nm^{-2}$ and volume $V_1 = (500- H)A$ , where A is the area of cross-section of the vessel.

Finally, the volume of air column above water is 300 A. If $P_2$ is the pressure of air then

$$P_2 + \rho g h = 10^5$$

$$\therefore \quad P_2 + 10^3 \times 10 \times \frac{200}{1000} = 10^5$$




$$\therefore \quad P_2 = 9.8 \times 10^4 N/m^2$$

As the temperature remains constant, according to Boyle's law

$P_1V_1 = P_2V_2$

∴ $10^5 \times (500 - H) A = (9.8 \times 10^4) \times 300A \Rightarrow H = 206$ mm

∴ The fall of height of water level due to the opening of orifice = 206 – 200 = 6 mm

**Q.3. A 0.1 kg mass is suspended from a wire of negligible mass.**

**The length of the wire is 1m and its crosssectional area is 4.9 × $10^{-7}$ $m^2$. If the mass is pulled a little in the vertically downward direction and released, it performs simple harmonic motion of angular frequency 140 rad $s^{-1}$. If the Young's modulus of the material of the wire is n × $10^9$ $Nm^{-2}$, the value of n is (2010)**

**Ans.** 4

**Solution.**

We know that $\omega = \sqrt{\frac{K}{m}}$ ...(i)

Here $Y = \frac{FL}{Al} \Rightarrow F = \left(\frac{YA}{L}\right)l$

**Comparing the above equation with F = kl we get**

$K = \left(\frac{YA}{L}\right)$ ...(ii)

From (i) & (ii), $\omega = \sqrt{\frac{YA}{ml}}$

$\therefore 140 = \sqrt{\frac{n \times 10^9 \times 4.9 \times 10^{-7}}{0.1 \times 1}} \quad \therefore n = 4$

**Q.4. Consider two solid spheres P and Q each of density 8 gm cm$^{-3}$ and diameters 1 cm and 0.5 cm, respectively. Sphere P is dropped into a liquid of density 0.8 gm cm–3 and viscosity η = 3 poiseulles. Sphere Q is dropped into a liquid of density 1.6 gm cm–3 and viscosity η = 2 poiseulles. The ratio of the terminal velocities of P and Q is (JEE Adv. 2016)**

**Ans.** 3

**Solution.**

$$\frac{V_P}{V_Q} = \frac{\frac{2r_1^2(\sigma - \rho_1)g}{9\eta_1}}{\frac{2r_2^2(\sigma - \rho_2)g}{9\eta_2}} = \frac{r_1^2(\sigma - \rho_1)}{r_2^2(\sigma - \rho_2)} \times \frac{\eta_2}{\eta_1}$$

$$= \frac{1^2}{(0.5)^2}\frac{[8 - 0.8]}{[8 - 1.6]} \times \frac{2}{3} = 3$$